Quantum Mechanics – Exercise Set 1: de Broglie equations

* Data:

Mass of electron me = 9.1 x 10-31 kg

Mass of proton mp =1.67 x 10-27 kg

Speed of light c in vacuum = 3 x 108 m/s

Planck’s constant h = 6.63 x 10-34 J-s

Reduced Planck’s constant ħ = h/2π = 1.055 x 10-34 J-s

Elementary charge q = 1.6 x 10-19 C

* Class-work:

1. Calculate de’Broglie wavelength of a subatomic particle of mass 200me moving with a speed of 0.98c.
2. Calculate de’Broglie wavelengths of (i) cricket ball of mass 450 gm thrown at a speed of 150 km/hr (ii) electron in a cathode ray tube (CRT) fired at a speed of 106 m/s. Comment on your results.
3. What is de’Broglie wavelength of a neutron having energy 1 MeV. Use mn = mp.
4. Calculate de’Broglie wavelength of a proton accelerated through 100 kV potential difference.
5. Through what potential difference must a deuteron be accelerated in order to have de’Broglie wavelength of 15 fm? Given md = 2mp and qd = q.
6. Find kinetic energy of an electron whose de’Broglie wavelength is the same as that of a photon possessing energy 100 keV.
7. An electron and a proton have the same de’Broglie wavelengths. Compare their kinetic energies. Given mp = 1800 me.
8. An electron and a muon (μ-) are accelerated through the same potential difference. How do their de’Broglie wavelengths compare? Given mµ = 207 me and qμ = qe = q
9. Estimate the minimum energy possessed by an electron in an atom using uncertainty principle.
10. What is the uncertainty in the determination of position of a particle if its momentum is measured to be 2 x 10-24 kg-m/s with an uncertainty of 0.05%?
11. Calculate uncertainty in the determination of momentum of an electron confined to a quantum well of size 1 nm. What is the percentage uncertainty in the momentum if its mean speed is 106 m/s?
12. Determine uncertainty in the measurement of momentum of a marble of mass 10 gm confined to a box of dimensions 50 cm. What is the percentage uncertainty in the momentum if it is moving with a speed of 20 cm/s. Is it significant as compared to the result of preceding example? What can you say about the measurement?
13. Calculate the percentage uncertainty in the measurement of momentum of a neutron having energy 20 MeV confined to a region of width equal to (i) 3 nuclei (ii) 3 atoms. Comment on the results.
14. The position of a proton is determined within an accuracy of 1 Å. Determine uncertainty in the measurement of its position 1 µs later.
15. The lifetime of an excited state of nucleus is usually 1 ps. Estimate uncertainty in energy of a γ-ray emitted by a nucleus.
16. Calculate the width of a spectral line if the transition giving rise to this spectral line has occurred during 0.01 µs seconds. What fraction it is of the frequencies of light in the visible spectrum?
17. Calculate the energy, momentum and de’ Broglie wavelength of an electron trapped in a one-dimensional quantum well of size 10 Å in its ground state.
18. Calculate the difference in energies of the first two allowed states for (i) electron confined to a strip of 10 nm and (ii) marble of mass 10 gm confined to a box of size 10 cm. What can you say about the results?
19. The wave function of a particle is given by Find the probability that the particle can be found between x = 0.45 to x = 0.55.
20. Determine the normalization constant for a particle whose wave function is given by in the interval x = 0 to L when L = 0.2.

* Answers to Class-Work:

(1) 1.24 x 10-14 m, (2) 3.54 x 10-35 m, this value is beyond any measurement. Hence, wave nature is not revealed at macroscopic level, 7.286 x 10-10 m, this value is verifiable experimentally. Hence, wave nature is significant at microscopic level, (3) 2.868 x 10-14 m, (4) 9.07 x 10-14 m, (5) 1.83 x 106 volt, (6) 9768 eV, (7) 1800:1, (8) 14.387:1, (9) 3.4 eV (10) ~ 0.1 micron, (11) 11.6%, (12) 1.055 x 10-30 %. Uncertainty in momentum is infinitesimally small and hence insignificant as compared to earlier example. Momentum is measured with perfect accuracy, (13) (i) 34%, (ii) 0.00034%. In first case, the position of neutron is almost accurate which leads to larger uncertainty in its momentum. In the second case, the position uncertainty is large, which effectively reduced the momentum uncertainty, (14) 0.83 mm (15) 0.00066 eV (16) 100 MHz, part of a million, (17) 0.377 eV, ±3.315 x 10-15 kg-m/s, 20 Å, (18) Electron: 0.011 eV; energies are discrete, Marble: ~ 10-45 eV; difference in energies are so small that energy can be regarded as continuous, (19) 3.9% (20) ± 2.69.